Chapter 5. Factorisation

Exercise 5(A)

Solution 1:

$3a^2 - 9ab = 3a \times a - 3a \times 3b$
$= 3a(a - 3b)$

Solution 2:

[Taking (x + y) common from both terms]

$= (x + y)[2(x + y)^2 - 6]$

$= 2(x + y)[(x + y)^2 - 3]$

$= 2(x + y)(x^2 + y^2 + 2xy - 3)$

Solution 3:

Taking (2x - 3y) common from both terms

$= (2x - 3y)[x^3 - x^2 (2x - 3y)]$

$= x^2(2x - 3y)[x - (2x - 3y)]$

$= x^2(2x - 3y)[x - 2x + 3y]$

$= x^2(2x - 3y)[-x + 3y]$

$= x^2(2x - 3y)(3y - x)$

Solution 4:

Taking (2x - 5y) common from both terms

$= (2x - 5y)[2(3x + 4y) - 6(x - y)]$

$= (2x - 5y)(6x + 8y - 6x + 6y)$

$= (2x - 5y)(8y + 6y)$

$= (2x - 5y)(14y)$

$= (2x - 5y)14y$

Solution 5:

$a^3 + a - 3a^2 - 3 = a(a^2 + 1) - 3(a^2 + 1)$
$= (a^2 + 1)(a - 3).$

Solution 6:

$16(a+b)^2 - 4a - 4b = 16(a+b)^2 - 4(a+b)$
$= 4(a+b)[4(a+b) - 1]$
$= 4(a+b)(4a+4b-1)$

Solution 7:

$$a^4 - 2a^3 - 4a + 8 = a^3(a-2) - 4(a-2)$$
$$= (a^3 - 4)(a-2)$$

Solution 8:

$$ab - 2b + a^2 - 2a = b(a-2) + a(a-2)$$
$$= (a+b)(a-2)$$

Solution 9:

$$ab(x^2+1) + x(a^2+b^2) = abx^2 + ab + a^2x + b^2x$$
$$= ax(bx+a) + b(bx+a)$$
$$= (ax+b)(bx+a)$$

Solution 10:

$$a^2 + b - ab - a = a^2 - a + b - ab$$
$$= a(a-1) + b(1-a)$$
$$= a(a-1) - b(a-1)$$
$$= (a-1)(a-b)$$

Solution 11:

$$(ax+by)^2 + (bx-ay)^2 = a^2x^2 + b^2y^2 + 2axby + b^2x^2 + a^2y^2 - 2bxay$$
$$= a^2x^2 + b^2y^2 + b^2x^2 + a^2y^2$$
$$= x^2(a^2+b^2) + y^2(a^2+b^2)$$
$$= (x^2+y^2)(a^2+b^2)$$

Solution 12:

$$a^2x^2 + (ax^2+1)x + a = a^2x^2 + a + (ax^2+1)x$$
$$= a(ax^2+1) + x(ax^2+1)$$
$$= (a+x)(ax^2+1)$$

Solution 13:

$$(2a-b)^2 - 10a + 5b = (2a-b)^2 - 5(2a-b)$$
$$= (2a-b)(2a-b-5)$$

Solution 14:

$$a(a-4) - a + 4 = a(a-4) - 1(a-4)$$
$$= (a-4)(a-1)$$

Solution 15:

$$y^2 - (a+b)y + ab = y^2 - ay - by + ab$$
$$= y(y-a) - b(y-a)$$
$$= (y-a)(y-b)$$

Solution 16:

$$a^2 + \frac{1}{a^2} - 2 - 3a + \frac{3}{a} = \left(a - \frac{1}{a}\right)^2 - 3\left(a - \frac{1}{a}\right)$$
$$= \left(a - \frac{1}{a}\right)\left[\left(a - \frac{1}{a}\right) - 3\right]$$
$$= \left(a - \frac{1}{a}\right)\left[a - \frac{1}{a} - 3\right]$$

Solution 17:

$= (x^2 + y^2 + 2xy) + (x + y)$

$[\text{As } (x + y)^2 = x^2 + 2xy + y^2]$

$=(x + y)^2 + (x + y)$

$=(x + y)(x + y + 1)$

Solution 18:

$= a^2 + 4b^2 - 4ab - 3a + 6b$

$= a^2 + (2b)^2 - 2 \times a \times (2b) - 3(a - 2b)$

$[\text{As } (a - b)^2 = a^2 - 2ab + b^2]$

$=(a - 2b)^2 - 3(a - 2b)$

$=(a - 2b)[(a - 2b) - 3]$

$=(a - 2b)(a - 2b - 3)$

Solution 19:

$= m(x-3y)^2 - n(x-3y) + 5(x-3y)$

[Taking (x - 3y) common from all the three terms]

$=(x-3y)[m(x-3y)-n+5]$

$=(x-3y)(mx-3my-n+5)$

Solution 20:

$=(6x-5y)[x-4(6x-5y)]$

[Taking (6x - 5y) common from the three terms]

$=(6x-5y)(x-24x+20y)$

$=(6x-5y)(-23x+20y)$

$=(6x-5y)(20y-23x)$

Exercise 5(B)

Solution 1:

$$\begin{aligned}a^2+10a+24 &= a^2+6a+4a+24\\ &= a(a+6)+4(a+6)\\ &= (a+6)(a+4)\end{aligned}$$

Solution 2:

$$\begin{aligned}a^2-3a-40 &= a^2-8a+5a-40\\ &= a(a-8)+5(a-8)\\ &= (a-8)(a+5)\end{aligned}$$

Solution 3:

$$\begin{aligned}1-2a-3a^2 &= 1-3a+a-3a^2\\ &= 1(1-3a)+a(1-3a)\\ &= (1+a)(1-3a)\end{aligned}$$

Solution 4:

$$\begin{aligned}x^2-3ax-88a^2 &= x^2-11ax+8ax-88a^2\\ &= x(x-11a)+8a(x-11a)\\ &= (x+8a)(x-11a)\end{aligned}$$

Solution 5:

$$\begin{aligned}6a^2 - a - 15 &= 6a^2 - 10a + 9a - 15\\ &= 2a(3a-5) + 3(3a-5)\\ &= (2a+3)(3a-5)\end{aligned}$$

Solution 6:

$$\begin{aligned}24a^3 + 37a^2 - 5a &= a(24a^2 + 37a - 5)\\ &= a(24a^2 + 40a - 3a - 5)\\ &= a \times [8a(3a+5) - 1(3a+5)]\\ &= a[(8a-1)(3a+5)]\\ &= a(8a-1)(3a+5)\end{aligned}$$

Solution 7:

$$\begin{aligned}a(3a-2) - 1 &= 3a^2 - 2a - 1\\ &= 3a^2 - 3a + a - 1\\ &= 3a(a-1) + 1(a-1)\\ &= (3a+1)(a-1)\end{aligned}$$

Solution 8:

$$\begin{aligned}a^2b^2 + 8ab - 9 &= a^2b^2 + 9ab - ab - 9\\ &= ab(ab+9) - 1(ab+9)\\ &= (ab+9)(ab-1)\end{aligned}$$

Solution 9:

$$\begin{aligned}3 - a(4+7a) &= 3 - 4a - 7a^2\\ &= 3 - 7a + 3a - 7a^2\\ &= 1(3-7a) + a(3-7a)\\ &= (3-7a)(a+1)\end{aligned}$$

Solution 10:

$(2a+b)^2 - 6a - 3b - 4 = (2a+b)^2 - 3(2a+b) - 4$

Assume that 2a+b=x

Therefore,

$$(2a+b)^2 - 6a - 3b - 4 = x^2 - 3x - 4$$
$$= x^2 - 4x + x - 4$$
$$= 1(x-4) + x(x-4)$$
$$= (x+1)(x-4)$$
$$= (2a+b+1)(2a+b-4)$$

[resubstitute the value of x]

Solution 11:

Assume that a + b = x;

$$1 - 2(a+b) - 3(a+b)^2 = 1 - 2x - 3x^2$$
$$= 1 - 3x + x - 3x^2$$
$$= 1(1-3x) + x(1-3x)$$
$$= (1-3x)(1+x)$$
$$= (1-3(a+b))(1+(a+b))$$
$$= (1-3a-3b)(1+a+b)$$

Solution 12:

$$3a^2 - 1 - 2a = 3a^2 - 2a - 1$$
$$= 3a^2 - 3a + a - 1$$
$$= 3a(a-1) + 1(a-1)$$
$$= (3a+1)(a-1)$$

Solution 13:

$$x^2 + 3x + 2 + ax + 2a = x^2 + 2x + x + 2 + ax + 2a$$
$$= x(x+2) + 1(x+2) + a(x+2)$$
$$= (x+2)(x+a+1)$$

Solution 14:

Assume that $3x - 2y = a$

Therefore,

$$\begin{aligned}(3x-2y)^2 + 3(3x-2y) - 10 &= a^2 + 3a - 10\\ &= a^2 + 5a - 2a - 10\\ &= a(a+5) - 2(a+5)\\ &= (a+5)(a-2)\\ &= (3x-2y+5)(3x-2y-2)\end{aligned}$$

Solution 15:

$$5-(3a^2-2a)(6-3a^2+2a) = 5-(3a^2-2a)\left[6-(3a^2-2a)\right]$$

Assume that $3a^2 - 2a = x$

Therefore,

$$\begin{aligned}5-(3a^2-2a)(6-3a^2+2a) &= 5 - x(6-x)\\ &= 5 - 6x + x^2\\ &= 5 - 5x - x + x^2\\ &= 5(1-x) - x(1-x)\\ &= (5-x)(1-x)\\ &= (x-5)(x-1)\\ &= (3a^2-2a-5)(3a^2-2a-1)\\ &= (3a^2-5a+3a-5)(3a^2-3a+a-1)\\ &= (a(3a-5)+1(3a-5))(3a(a-1)+1(a-1))\\ &= (3a-5)(a+1)(3a+1)(a-1)\end{aligned}$$

Solution 16:

(i) Given expression: $x^2 - 3x - 54$

Comparing with $ax^2 + bx + c$, we get $a = 1$, $b = -3$ and $c = -54$

$\therefore b^2 - 4ac = (-3)^2 - 4(1)(-54) = 9 + 216 = 225$, which is a perfect square.

$\therefore x^2 - 3x - 54$ is factorisable.

$$\begin{aligned} \text{Now, } x^2 - 3x - 54 &= x^2 - 9x + 6x - 54 \\ &= x(x-9) + 6(x-9) \\ &= (x-9)(x+6) \end{aligned}$$

(ii) Given expression: $2x^2 - 7x - 15$

Comparing with $ax^2 + bx + c$, we get $a = 2$, $b = -7$ and $c = -15$

$\therefore b^2 - 4ac = (-7)^2 - 4(2)(-15) = 49 + 120 = 169$, which is a perfect square.

$\therefore 2x^2 - 7x - 15$ is factorisable.

$$\begin{aligned} \text{Now, } 2x^2 - 7x - 15 &= 2x^2 - 10x + 3x - 15 \\ &= 2x(x-5) + 3(x-5) \\ &= (2x+3)(x-5) \end{aligned}$$

(iii) Given expression: $2x^2 + 2x - 75$

Comparing with $ax^2 + bx + c$, we get $a = 2$, $b = 2$ and $c = -75$

$\therefore b^2 - 4ac = (2)^2 - 4(2)(-75) = 4 + 600 = 604$, which is not a perfect square.

$\therefore 2x^2 + 2x - 75$ is not factorisable.

(iv) Given expression: $3x^2 + 4x - 10$

Comparing with $ax^2 + bx + c$, we get $a = 3$, $b = 4$ and $c = -10$

$\therefore b^2 - 4ac = (4)^2 - 4(3)(-10) = 16 + 120 = 136$, which is not a perfect square.

$\therefore 3x^2 + 4x - 10$ is not factorisable.

(v) Given expression: $x(2x-1) - 1$

Now, $x(2x-1) - 1 = 2x^2 - x - 1$

Comparing with $ax^2 + bx + c$, we get $a = 2$, $b = -1$ and $c = -1$

$\therefore b^2 - 4ac = (-1)^2 - 4(2)(-1) = 1 + 8 = 9$, which is a perfect square.

$\therefore 2x^2 - x - 1$ is factorisable.

$$\begin{aligned} \text{Now, } 2x^2 - x - 1 &= 2x^2 - 2x + x - 1 \\ &= 2x(x-1) + 1(x-1) \\ &= (2x+1)(x-1) \end{aligned}$$

Exercise 5(C)

Solution 1:

$$\begin{aligned} 25a^2 - 9b^2 &= (5a)^2 - (3b)^2 \\ &= (5a - 3b)(5a + 3b) \quad [\because a^2 - b^2 = (a+b)(a-b)] \end{aligned}$$

Solution 2:

$$a^2-(2a+3b)^2=(a)^2-(2a+3b)^2$$
$$=(a-2a-3b)(a+2a+3b)\ [\because a^2-b^2=(a+b)(a-b)]$$
$$=(-a-3b)(3a+3b)$$
$$=-3(a+3b)(a+b)$$

Solution 3:

$$a^2-81(b-c)^2=(a)^2-[9(b-c)]^2$$
$$=(a-(9b-9c))(a+(9b-9c))\ [\because a^2-b^2=(a+b)(a-b)]$$
$$=(a-9b+9c)(a+9b-9c)$$

Solution 4:

$$25(2a-b)^2-81b^2=[5(2a-b)]^2-(9b)^2$$
$$=[5(2a-b)-9b][5(2a-b)+9b]$$
$$[\because a^2-b^2=(a+b)(a-b)]$$
$$=[10a-5b-9b][10a-5b+9b]$$
$$=[10a-14b][10a+4b]$$
$$=2\times(5a-7b)\times 2\times(5a+2b)$$
$$=4(5a-7b)(5a+2b)$$

Solution 5:

$$50a^3-2a=2a(25a^2-1)$$
$$=2a\left[(5a)^2-1^2\right]$$
$$=2a(5a+1)(5a-1)\quad[\because a^2-b^2=(a+b)(a-b)]$$

Solution 6:

$$4a^2b-9b^3=b(4a^2-9b^2)$$
$$=b\left[(2a)^2-(3b)^2\right]$$
$$=b(2a-3b)(2a+3b)\quad[\because a^2-b^2=(a+b)(a-b)]$$

Solution 7:

$$3a^5-108a^3=3a^3(a^2-36)$$
$$=3a^3\left[(a)^2-(6)^2\right]$$
$$=3a^3(a-6)(a+6)\qquad\because a^2-b^2=(a+b)(a-b)$$

Solution 8:

$$9(a-2)^2-16(a+2)^2=[3(a-2)]^2-[4(a+2)]^2$$
$$=[3(a-2)-4(a+2)][3(a-2)+4(a+2)]$$
$$[\because a^2-b^2=(a+b)(a-b)]$$
$$=[3a-6-4a-8][3a-6+4a+8]$$
$$=(-a-14)(7a+2)$$
$$=-(a+14)(7a+2)$$

Solution 9:

$$a^4-1=(a^2)^2-(1)^2$$
$$=(a^2+1)(a^2-1)\ [\because a^2-b^2=(a+b)(a-b)]$$
$$=(a^2+1)((a)^2-(1)^2)$$
$$=(a^2+1)(a+1)(a-1)$$

Solution 10:

$$a^3+2a^2-a-2=a^2(a+2)-1(a+2)$$
$$=(a^2-1)(a+2)$$
$$=(a+1)(a-1)(a+2)\quad[\because a^2-b^2=(a+b)(a-b)]$$

Solution 11:

$$(a+b)^3-a-b=(a+b)^3-(a+b)$$
$$=(a+b)\left[(a+b)^2-1\right]$$
$$=(a+b)\left[(a+b)^2-1^2\right]$$
$$=(a+b)((a+b)+1)((a+b)-1)$$
$$[\because a^2-b^2=(a+b)(a-b)]$$
$$=(a+b)(a+b+1)(a+b-1)$$

Solution 12:

$$a(a-1)-b(b-1)=a^2-a-b^2+b$$
$$=a^2-b^2-a+b$$
$$=(a+b)(a-b)-(a-b)$$
$$[\because a^2-b^2=(a+b)(a-b)]$$
$$=(a-b)[(a+b)-1]$$
$$=(a-b)[a+b-1]$$

Solution 13:

$$4a^2 - (4b^2 + 4bc + c^2) = (2a)^2 - (2b + c)^2$$
$$= [2a - (2b + c)][2a + (2b + c)]$$
$$[\because a^2 - b^2 = (a + b)(a - b)]$$
$$= [2a - 2b - c][2a + 2b + c]$$

Solution 14:

$$4a^2 - 49b^2 + 2a - 7b = [(2a)^2 - (7b)^2] + [2a - 7b]$$
$$= [2a - 7b]\,[2a + 7b] + [2a - 7b]$$
$$[\because a^2 - b^2 = (a + b)(a - b)]$$
$$= [2a - 7b]\,[2a + 7b + 1]$$

Solution 15:

$$9a^2 + 3a - 8b - 64b^2 = 9a^2 - 64b^2 + 3a - 8b$$
$$= (3a)^2 - (8b)^2 + 3a - 8b$$
$$= (3a - 8b)(3a + 8b) + (3a - 8b)$$
$$[\because a^2 - b^2 = (a + b)(a - b)]$$
$$= (3a - 8b)(3a + 8b + 1)$$

Solution 16:

$$4a^2 - 12a + 9 - 49b^2 = (2a)^2 - 12a + (3)^2 - 49b^2$$
$$= (2a - 3)^2 - 49b^2$$
$$= (2a - 3)^2 - (7b)^2$$
$$= (2a - 3 - 7b)(2a - 3 + 7b)$$
$$[\because a^2 - b^2 = (a + b)(a - b)]$$

Solution 17:

$$4xy - x^2 - 4y^2 + z^2 = z^2 - (x^2 + 4y^2 - 4xy)$$
$$= z^2 - (x - 2y)^2$$
$$= [z - (x - 2y)]\,[z + (x - 2y)]$$
$$[\because a^2 - b^2 = (a + b)(a - b)]$$
$$= [z - x + 2y][z + x - 2y]$$

Solution 18:

$$a^2+b^2-c^2-d^2+2ab-2cd$$
$$=(a^2+b^2+2ab)-(c^2+d^2+2cd)$$
$$=(a+b)^2-(c+d)^2$$
$$=[(a+b)-(c+d)][(a+b)+(c+d)] \quad [\because a^2-b^2=(a+b)(a-b)]$$
$$=(a+b-c-d)(a+b+c+d)$$

Solution 19:

$$4x^2-12ax-y^2-z^2-2yz+9a^2$$
$$=4x^2+9a^2-12ax-y^2-z^2-2yz$$

$$=(2x)^2+(3a)^2-12ax-(y^2+z^2+2yz)$$
$$=(2x-3a)^2-(y+z)^2$$
$$=[(2x-3a)-(y+z)][(2x-3a)+(y+z)]$$
$$[\because a^2-b^2=(a+b)(a-b)]$$
$$=[2x-3a-y-z][2x-3a+y+z]$$

Solution 20:

$$(a^2-1)(b^2-1)+4ab=a^2b^2-a^2-b^2+1+4ab$$
$$=a^2b^2+1+2ab-a^2-b^2+2ab$$
$$=(a^2b^2+1+2ab)-(a^2+b^2-2ab)$$
$$=(ab+1)^2-(a-b)^2$$
$$=[(ab+1)-(a-b)][(ab+1)+(a-b)]$$
$$[\because a^2-b^2=(a+b)(a-b)]$$
$$=[ab+1-a+b][ab+1+a-b]$$

Solution 21:

$$x^4+x^2+1=x^4+2x^2+1-x^2$$
$$=(x^2)^2+2x^2+(1)^2-x^2$$
$$=(x^2+1)^2-(x)^2$$
$$[\because a^2-b^2=(a+b)(a-b)]$$
$$=(x^2+1-x)(x^2+1+x)$$

Solution 22:

$$\left(a^2+b^2-4c^2\right)^2-4a^2b^2=\left(a^2+b^2-4c^2\right)^2-(2ab)^2$$
$$=\left(a^2+b^2-4c^2-2ab\right)\left(a^2+b^2-4c^2+2ab\right)$$
$$\left[\because a^2-b^2=(a+b)(a-b)\right]$$
$$=\left(a^2+b^2-2ab-4c^2\right)\left(a^2+b^2+2ab-4c^2\right)$$
$$=\left((a-b)^2-(2c)^2\right)\left((a+b)^2-(2c)^2\right)$$
$$=(a-b+2c)(a-b-2c)(a+b+2c)(a+b-2c)$$

Solution 23:

$$\left(x^2+4y^2-9z^2\right)^2-16x^2y^2$$
$$=\left(x^2+4y^2-9z^2\right)^2-(4xy)^2$$
$$=\left(x^2+4y^2-9z^2-4xy\right)\left(x^2+4y^2-9z^2+4xy\right)$$
$$\left[\because a^2-b^2=(a+b)(a-b)\right]$$
$$=\left(x^2+4y^2-4xy-9z^2\right)\left(x^2+4y^2+4xy-9z^2\right)$$
$$=\left[(x-2y)^2-(3z)^2\right]\left[(x+2y)^2-(3z)^2\right]$$
$$=\left[(x-2y)-3z\right]\left[(x-2y)+3z\right]\left[(x+2y)-3z\right]\left[(x+2y)+3z\right]$$
$$=\left[x-2y-3z\right]\left[x-2y+3z\right]\left[x+2y-3z\right]\left[x+2y+3z\right]$$

Solution 24:

$$(a+b)^2-a^2+b^2$$
$$=a^2+2ab+b^2-a^2+b^2$$
$$=2ab+2b^2$$
$$=2b(a+b)$$

Solution 25:

$$a^2-b^2-(a+b)^2$$
$$=a^2-b^2-(a^2+2ab+b^2)$$
$$=a^2-b^2-a^2-2ab-b^2$$
$$=-2ab-2b^2$$
$$=-2b(a+b)$$

Solution 26:

$$9a^2 - (a^2 - 4)^2$$
$$= (3a)^2 - (a^2 - 4)^2$$
$$= [3a - (a^2 - 4)][3a + (a^2 - 4)]$$
$$= [3a - a^2 - 4][3a + a^2 - 4]$$
$$= [-a^2 + 3a - 4][a^2 + 3a - 4]$$
$$= [-a^2 + 4a - a - 4][a^2 + 4a - a - 4]$$
$$= [a(-a + 4) + 1(-a + 4)][a(a + 4) - 1(a + 4)]$$
$$= [(a + 1)(4 - a)][(a + 4)(a - 1)]$$
$$= (a + 1)(4 - a)(a + 4)(a - 1)$$

Solution 27:

$$x^2 + \frac{1}{x^2} - 11$$
$$= x^2 + \frac{1}{x^2} - 2 - 9$$
$$= x^2 + \frac{1}{x^2} - 2 \times x \times \frac{1}{x} - 9$$
$$= \left(x - \frac{1}{x}\right)^2 - (3)^2$$
$$= \left(x - \frac{1}{x} + 3\right)\left(x - \frac{1}{x} - 3\right)$$

Solution 28:

$$4x^2 + \frac{1}{4x^2} + 1$$
$$= 4x^2 + \frac{1}{4x^2} + 2 - 1$$
$$= 4x^2 + \frac{1}{4x^2} + 2 \times 2x \times \frac{1}{2x} - 1$$
$$= \left(2x + \frac{1}{2x}\right)^2 - (1)^2$$
$$= \left(2x + \frac{1}{2x} + 1\right)\left(2x + \frac{1}{2x} - 1\right)$$

Solution 29:

$4x^4 - x^2 - 12x - 36$
$= 4x^4 - (x^2 + 12x + 36)$
$= (2x^2)^2 - (x^2 + 2 \times x \times 6 + 6^2)$
$= (2x^2)^2 - (x + 6)^2$
$= (2x^2 + x + 6)(2x^2 - x - 6)$
$= (2x^2 + x + 6)(2x^2 - 4x + 3x - 6)$
$= (2x^2 + x + 6)[2x(x - 2) + 3(x - 2)]$
$= (2x^2 + x + 6)[(x - 2)(2x + 3)]$
$= (2x^2 + x + 6)(x - 2)(2x + 3)$

Solution 30:

$a^2(b + c) - (b + c)^3$
$= (b + c)[a^2 - (b + c)^2]$
$= (b + c)[(a + b + c)(a - b - c)]$
$= (b + c)(a + b + c)(a - b - c)$

Exercise 5(D)

Solution 1:

$a^3 - 27 = (a)^3 - (3)^3$
$= (a - 3)\left[(a)^2 + a \times 3 + (3)^2\right] \quad \left[\because a^3 - b^3 = (a - b)(a^2 + ab + b^2)\right]$
$= (a - 3)\left[a^2 + 3a + 9\right]$

Solution 2:

$1 - 8a^3 = (1)^3 - (2a)^3$
$= (1 - 2a)\left[(1)^2 + 1 \times 2a + (2a)^2\right]$
$\left[\because a^3 - b^3 = (a - b)(a^2 + ab + b^2)\right]$
$= (1 - 2a)\left[1 + 2a + 4a^2\right]$

Solution 3:

$64 - a^3b^3 = (4)^3 - ab^3$
$= (4 - ab)\left[(4)^2 + 4 \times ab + (ab)^2\right]$
$\left[\because a^3 - b^3 = (a - b)(a^2 + ab + b^2)\right]$
$= (4 - ab)\left[16 + 4ab + a^2b^2\right]$

Solution 4:

$$a^6 + 27b^3 = (a^2)^3 + (3b)^3$$
$$= (a^2 + 3b)\left[(a^2)^2 - a^2 \times 3b + (3b)^2\right]$$
$$\left[\because a^3 + b^3 = (a+b)(a^2 - ab + b^2)\right]$$
$$= (a^2 + 3b)\left[a^4 - 3a^2b + 9b^2\right]$$

Solution 5:

$$3x^7y - 81x^4y^4 = 3xy\left(x^6 - 27x^3y^3\right)$$
$$= 3xy\left((x^2)^3 - (3xy)^3\right)$$
$$= 3xy\left(x^2 - 3xy\right)\left[(x^2)^2 + x^2 \times 3xy + (3xy)^2\right]$$
$$\left[\because a^3 - b^3 = (a-b)(a^2 + ab + b^2)\right]$$
$$= 3xy\left(x^2 - 3xy\right)\left[x^4 + 3x^3y + 9x^2y^2\right]$$
$$= 3xy\left\{x(x - 3y)x^2\left[x^2 + 3xy + 9y^2\right]\right\}$$
$$= 3x^4y(x - 3y)\left[x^2 + 3xy + 9y^2\right]$$

Solution 6:

$$a^3 - \frac{27}{a^3} = (a)^3 - \left(\frac{3}{a}\right)^3$$
$$= \left(a - \frac{3}{a}\right)\left(a^2 + a \times \frac{3}{a} + \left(\frac{3}{a}\right)^2\right)$$
$$\left[\because a^3 - b^3 = (a-b)(a^2 + ab + b^2)\right]$$
$$= \left(a - \frac{3}{a}\right)\left(a^2 + 3 + \frac{9}{a^2}\right)$$

Solution 7:

$$a^3 + 0.064 = (a)^3 + (0.4)^3$$
$$= (a + 0.4)\left[(a)^2 - a \times 0.4 + (0.4)^2\right]$$
$$\left[\because a^3 + b^3 = (a+b)(a^2 - ab + b^2)\right]$$
$$= (a + 0.4)\left[a^2 - 0.4a + 0.16\right]$$

Solution 8:

$$a^4 - 343a = a(a^3 - 7^3)$$
$$= a(a-7)\left[(a)^2 + a \times 7 + (7)^2\right]$$
$$\left[\because a^3 - b^3 = (a-b)(a^2 + ab + b^2)\right]$$
$$= a(a-7)\left[a^2 + 7a + 49\right]$$

Solution 9:

$= (x - y)^3 - (2x)^3$

$= (x - y - 2x)[(x - y)^2 + 2x(x - y) + (2x)^2]$

[Using identity $(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$]

$= (-x - y)[x^2 + y^2 - 2xy + 2x^2 - 2xy + 4x^2]$

$=-(x + y)\ [7x^2 - 4xy + y^2]$

Solution 10:

$$\frac{8a^3}{27} - \frac{b^3}{8} = \left(\frac{2a}{3}\right)^3 - \left(\frac{b}{2}\right)^3$$
$$= \left(\frac{2a}{3} - \frac{b}{2}\right)\left[\left(\frac{2a}{3}\right)^2 + \frac{2a}{3} \times \frac{b}{2} + \left(\frac{b}{2}\right)^2\right]$$
$[\because a^3 - b^3 = (a - b)(a^2 = ab + b^2)]$
$$= \left(\frac{2a}{3} - \frac{b}{2}\right)\left[\frac{4a^2}{9} + \frac{ab}{3} + \frac{b^2}{4}\right]$$

Solution 11:

We know that,

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$ ....(1)

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$ ....(2)

$a^6 - b^6 = (a^3)^2 - (b^3)^2$

$= (a^3 + b^3)(a^3 - b^3)$

$= (a+b)(a^2 - ab + b^2)(a-b)(a^2 + ab + b^2)$ [from (1) and (2)]

$= (a+b)(a-b)(a^2 - ab + b^2)(a^2 + ab + b^2)$

Solution 12:

We know that,

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$ ....(1)

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$ ....(2)

$a^6 - 7a^3 - 8 = a^6 - 8a^3 + a^3 - 8$

$= a^3(a^3 - 8) + 1(a^3 - 8)$

$= (a^3 + 1)(a^3 - 8)$

$= (a^3 + 1^3)(a^3 - 2^3)$

$= (a+1)(a^2 - a + 1)(a-2)(a^2 + 2a + 4)$ [from (1) and (2)]

$= (a+1)(a-2)(a^2 - a + 1)(a^2 + 2a + 4)$

Solution 13:

We know that,

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$ ....(1)

$a^3 - 27b^3 + 2a^2b - 6ab^2$

$= (a)^3 - (3b)^3 + 2ab(a - 3b)$

$= (a - 3b)\left[a^2 + a \times 3b + (3b)^2\right] + 2ab(a - 3b)$ [from (1)]

$= (a - 3b)\left[a^2 + 3ab + 9b^2\right] + 2ab(a - 3b)$

$= (a - 3b)\left[a^2 + 3ab + 9b^2 + 2ab\right]$

$= (a - 3b)\left[a^2 + 5ab + 9b^2\right]$

Solution 14:

We know that,

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$ ....(1)

$8a^3 - b^3 - 4ax + 2bx = \left[(2a)^3 - (b)^3\right] - 2x(2a - b)$

$= (2a - b)\left[(2a)^2 + 2a \times b + (b)^2\right] - 2x(2a - b)$

[from (1)]

$= (2a - b)\left[4a^2 + 2ab + b^2\right] - 2x(2a - b)$

$= (2a - b)\left[4a^2 + 2ab + b^2 - 2x\right]$

Solution 15:

We know that,

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2) \quad ....(1)$$

$$a - b - a^3 + b^3 = a - b - (a^3 - b^3)$$

$$= (a-b) - (a-b)\left[a^2 + ab + b^2\right] \quad [\text{from } (1)]$$

$$= (a-b)\left[1 - a^2 - ab - b^2\right]$$

Solution 16:

$= 2(x^3 + 27y^3 - 2x - 6y)$

$= 2\{[(x)^3 + (3y)^3] - 2(x + 3y)\}$

[Using identity $(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$]

$=2\{[(x + 3y)(x^2 - 3xy + 9y^2)] - 2(x + 3y)\}$

$=2(x + 3y)(x^2 - 3xy + 9y^2 - 2)$

Solution 17:

(i) $(13^3 - 5^3)$

[Using identity $(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$]

$=(13 - 5)(13^2 + 13 \times 5 + 5^2)$

$=8(169 + 65 + 25)$

Therefore, the number is divisible by 8.

(ii) $(35^3 + 27^3)$

[Using identity $(a^3 + b^3)=(a + b)(a^2 - ab + b^2)$]

$=(35 + 27)(35^2 + 35\times 27 + 27^2)$

$=62 \times (35^2 + 35 \times 27 + 27^2)$

Therefore, the number is divisible by 62.

Exercise 5(E)

Solution 1:

$$x^2+\frac{1}{4x^2}+1-7x-\frac{7}{2x}=(x)^2+\frac{1}{(2x)^2}+2\times x\times\frac{1}{2x}-7\left(x+\frac{1}{2x}\right)$$
$$=\left(x+\frac{1}{2x}\right)^2-7\left(x+\frac{1}{2x}\right)$$
$$=\left(x+\frac{1}{2x}\right)\left(x+\frac{1}{2x}-7\right)$$
$$=\left(x+\frac{1}{2x}\right)\left(x-7+\frac{1}{2x}\right)$$

Solution 2:

$$9a^2+\frac{1}{9a^2}-2-12a+\frac{4}{3a}=(3a)^2+\frac{1}{(3a)^2}-2\times 3a\times\frac{1}{3a}-4\left(3a-\frac{1}{3a}\right)$$
$$=\left(3a-\frac{1}{3a}\right)^2-4\left(3a-\frac{1}{3a}\right)$$
$$=\left(3a-\frac{1}{3a}\right)\left(\left(3a-\frac{1}{3a}\right)-4\right)$$
$$=\left(3a-\frac{1}{3a}\right)\left(3a-4-\frac{1}{3a}\right)$$

Solution 3:

$$x^2+\frac{a^2+1}{a}x+1=x^2+ax+\frac{1}{a}x+1$$
$$=x(x+a)+\frac{1}{a}(x+a)$$
$$=(x+a)\left(x+\frac{1}{a}\right)$$

Solution 4:

$$x^4+y^4-27x^2y^2=\left(x^2\right)^2+\left(y^2\right)^2-2x^2y^2-25x^2y^2$$
$$=\left(x^2-y^2\right)^2-25x^2y^2$$
$$=\left(x^2-y^2\right)^2-(5xy)^2\quad\left[\because a^2-b^2=(a+b)(a-b)\right]$$
$$=\left[\left(x^2-y^2\right)+5xy\right]\left[\left(x^2-y^2\right)-5xy\right]$$
$$=\left[x^2+5xy-y^2\right]\left[x^2-5xy-y^2\right]$$

Solution 5:

$$4x^4 + 9y^4 + 11x^2y^2 = (2x^2)^2 + (3y^2)^2 + 12x^2y^2 - x^2y^2$$
$$= (2x^2 + 3y^2)^2 - x^2y^2$$
$$= (2x^2 + 3y^2)^2 - (xy)^2$$
$$= (2x^2 + 3y^2 - xy)(2x^2 + 3y^2 + xy)$$
$$[\because a^2 - b^2 = (a+b)(a-b)]$$

Solution 6:

$$x^2 + \frac{1}{x^2} - 3 = x^2 + \frac{1}{x^2} - 2 - 1$$
$$= x^2 + \frac{1}{x^2} - 2 \times x \times \frac{1}{x} - 1$$
$$= \left(x - \frac{1}{x}\right)^2 - 1$$
$$= \left(x - \frac{1}{x}\right)^2 - (1)^2$$
$$= \left(x - \frac{1}{x} - 1\right)\left(x - \frac{1}{x} + 1\right) \quad [\because a^2 - b^2 = (a+b)(a-b)]$$

Solution 7:

$$a - b - 4a^2 + 4b^2 = (a-b) - 4(a^2 - b^2)$$
$$= (a-b) - 4(a-b)(a+b) \; [\because a^2 - b^2 = (a+b)(a-b)]$$
$$= (a-b)[1 - 4(a+b)]$$
$$= (a-b)[1 - 4a - 4b]$$

Solution 8:

$$(2a-3)^2 - 2(2a-3)(a-1) + (a-1)^2$$
$$= [(2a-3) - (a-1)]^2$$
$$= [2a - 3 - a + 1]^2$$
$$= (a-2)^2$$

Solution 9:

Let us assume, $a^2 - 3a = x$

Then the given expression is,

$$\begin{aligned}(a^2 - 3a)(a^2 - 3a + 7) + 10 &= x(x + 7) + 10\\ &= x^2 + 7x + 10\\ &= x^2 + 5x + 2x + 10\\ &= x(x + 5) + 2(x + 5)\\ &= (x + 5)(x + 2)\\ &= (a^2 - 3a + 5)(a^2 - 3a + 2)\\ &\quad [\text{resubstitute the value of } x]\\ &= (a^2 - 3a + 5)(a^2 - 2a - a + 2)\\ &= (a^2 - 3a + 5)(a(a - 2) - 1(a - 2))\\ &= (a^2 - 3a + 5)[(a - 1)(a - 2)]\end{aligned}$$

Solution 10:

Let us assume $a^2 - a = x$

Then the given expression is

$$\begin{aligned}(a^2 - a)(4a^2 - 4a - 5) - 6 &= x(4x - 5) - 6\\ &= 4x^2 - 5x - 6\\ &= 4x^2 - 8x + 3x - 6\\ &= 4x(x - 2) + 3(x - 2)\\ &= (4x + 3)(x - 2)\\ &= (4(a^2 - a) + 3)(a^2 - a - 2)\\ &\quad [\text{resubstitute the value of } x]\\ &= (4a^2 - 4a + 3)(a^2 - a - 2)\\ &= (4a^2 - 4a + 3)(a^2 - 2a + a - 2)\\ &= (4a^2 - 4a + 3)(a(a - 2) + 1(a - 2))\\ &= (4a^2 - 4a + 3)(a - 2)(a + 1)\end{aligned}$$

Solution 11:

$$\begin{aligned}x^4 + y^4 - 3x^2y^2 &= x^4 + y^4 - 2x^2y^2 - x^2y^2\\ &= (x^2)^2 + (y^2)^2 - 2x^2y^2 - x^2y^2\\ &= (x^2 - y^2)^2 - (xy)^2\\ &= (x^2 - y^2 - xy)(x^2 - y^2 + xy)\\ &\quad [\because a^2 - b^2 = (a + b)(a - b)]\end{aligned}$$

Solution 12:

$5a^2 - b^2 - 4ab + 7a - 7b$

$= 4a^2 + a^2 - b^2 - 4ab + 7a - 7b$

$= a^2 - b^2 + 4a^2 - 4ab + 7a - 7b$

$= (a^2 - b^2) + 4a(a - b) + 7(a - b)$

$= (a - b)(a + b) + 4a(a - b) + 7(a - b) \quad [\because a^2 - b^2 = (a + b)(a - b)]$

$= (a - b)[(a + b) + 4a + 7]$

$= (a - b)[a + b + 4a + 7]$

$= (a - b)[5a + b + 7]$

Solution 13:

$12(3x - 2y)^2 - 3x + 2y - 1 = 12(3x - 2y)^2 - (3x - 2y) - 1$

Let us assume that $3x - 2y = a$

Then the given expression is

$12(3x - 2y)^2 - 3x + 2y - 1 = 12a^2 - 3a - 1$

$= 12a^2 - 4a + 3a - 1$

$= 4a(3a - 1) + 1(3a - 1)$

$= (4a + 1)(3a - 1)$

$= (4(3x - 2y) + 1)(3(3x - 2y) - 1)$

[resubstitute the value of a]

$= (12x - 8y + 1)(9x - 6y - 1)$

Solution 14:

$4(2x - 3y)^2 - 8x + 12y - 3 = 4(2x - 3y)^2 - 4(2x - 3y) - 3$

Let us assume that $2x - 3y = a$

Then the given expression is

$4(2x - 3y)^2 - 8x + 12y - 3 = 4a^2 - 4a - 3$

$= 4a^2 - 6a + 2a - 3$

$= 2a(2a - 3) + 1(2a - 3)$

$= (2a - 3)(2a + 1)$

$= (2(2x - 3y) - 3)(2(2x - 3y) + 1)$

$= (4x - 6y - 3)(4x - 6y + 1)$

Solution 15:

$$3-5x+5y-12(x-y)^2 = 3-5(x-y)-12(x-y)^2$$

Let us assume that $x-y=a$

Then the given expression is

$$\begin{aligned}3-5x+5y-12(x-y)^2 &= 3-5a-12a^2\\ &= 3-9a+4a-12a^2\\ &= 3(1-3a)+4a(1-3a)\\ &= (3+4a)(1-3a)\\ &\quad [\text{resubstitute the value of a}]\\ &= (3+4(x-y))(1-3(x-y))\\ &= (3+4x-4y)(1-3x+3y)\end{aligned}$$

Solution 16:

$$\begin{aligned}&9x^2+3x-8y-64y^2\\ &= 9x^2-64y^2+3x-8y\\ &= [(3x)^2-(8y)^2]+(3x-8y)\\ &= [(3x+8y)(3x-8y)]+(3x-8y)\\ &= (3x-8y)(3x+8y+1)\end{aligned}$$

Solution 17:

$$\begin{aligned}&2\sqrt{3}x^2+x-5\sqrt{3}\\ &= 2\sqrt{3}x^2+6x-5x-5\sqrt{3}\\ &= 2\sqrt{3}x\left(x+\sqrt{3}\right)-5\left(x+\sqrt{3}\right)\\ &= \left(2\sqrt{3}x-5\right)\left(x+\sqrt{3}\right)\end{aligned}$$

Solution 18:

$$\begin{aligned}&\frac{1}{4}(a+b)^2-\frac{9}{16}(2a-b)^2\\ &= \frac{1}{4}\left[(a+b)^2-\frac{9}{4}(2a-b)^2\right]\\ &= \frac{1}{4}\left[(a+b)^2-\left(\frac{3}{2}(2a-b)\right)^2\right]\\ &= \frac{1}{4}\left[\left(a+b+\frac{3}{2}(2a-b)\right)\left(a+b-\frac{3}{2}(2a-b)\right)\right]\\ &= \frac{1}{4}\left[\left(a+b+3a-\frac{3b}{2}\right)\left(a+b-3a+\frac{3b}{2}\right)\right]\\ &= \frac{1}{4}\left[\left(4a-\frac{b}{2}\right)\left(\frac{5b}{2}-2a\right)\right]\\ &= \frac{1}{4}\left[\left(\frac{8a-b}{2}\right)\left(\frac{5b-4a}{2}\right)\right]\\ &= \frac{1}{4}\left[\frac{1}{4}(8a-b)(5b-4a)\right]\\ &= \frac{1}{16}(8a-b)(5b-4a)\end{aligned}$$

Solution 19:

$$2(ab + cd) - a^2 - b^2 + c^2 + d^2$$
$$= 2ab + 2cd - a^2 - b^2 + c^2 + d^2$$
$$= c^2 + d^2 + 2cd - a^2 - b^2 + 2ab$$
$$= (c^2 + d^2 + 2cd) - (a^2 + b^2 - 2ab)$$
$$= (c + d)^2 - (a - b)^2$$
$$= (c + d + a - b)(c + d - a + b)$$

Solution 20:

(i) $(987)^2 - (13)^2$

$$= (987 + 13)(987 - 13)$$
$$= 1000 \times 974$$
$$= 974000$$

(ii) $(67.8)^2 - (32.2)^2$

$$= (67.8 + 32.2)(67.8 - 32.2)$$
$$= 100 \times 35.6$$
$$= 3560$$

(iii) $\dfrac{(6.7)^2 - (3.3)^2}{6.7 - 3.3}$

$$= \frac{(6.7 + 3.3)(6.7 - 3.3)}{(6.7 - 3.3)}$$
$$= 10$$

(iv) $\dfrac{(18.5)^2 - (6.5)^2}{18.5 + 6.5}$

$$= \frac{(18.5 + 6.5)(18.5 - 6.5)}{(18.5 + 6.5)}$$
$$= 12$$